EXPERIMENTAL DATA ON CREEP OF ENGINEERING ALLOYS AND PHENOMENOLOGICAL THEORIES OF CREEP. A REVIEW.

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Zhurnal prikladnoi mekhaniki i tekhnicheskoi fiziki, No. 1, pp. 141-159, 1965

1. The phenomenon of creep of metals and alloys is primarily of interest because creep limits the useful life of many of the principal components of turbines, aircraft, chemical plants, etc.

On the one hand, we have the process of improving materials and developing new alloys with increased strength at high temperatures. This, and particularly the problem of developing heat-resistant alloys, is now attracting the attention of considerable numbers of metallurgists and metal physicists.

On the other hand, the designer must know how to use existing materials properly, i.e., how to estimate with accuracy the life of the part he is designing, dimension it correctly, choose the most suitable material available, and make the best use of its potentialities.

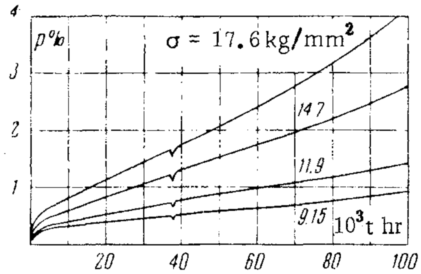

Fig. 1.

So far, physicists have played only a small part in solving this problem, which is primarily the concern of engineers and experts in the mechanics of solids concerned with developing phenomenological theories of creep. A phenomenological or mechanical theory correlates the results of macro-experiments and is formulated in terms of the mechanics of solids; the equations of such theories are formally constructed and do not take into account the micro-effects that condition the process, although many of their elements essentially describe definite micro-mechanisms in a summary way; in a number of cases, apparently, a direct correspondence can be established.

More than 50 years have passed since the first engineering investigations of the creep of metals; during this time an enormous amount of usually rather complex experimental material on various metals and alloys has been accumulated. This is because creep experiments usually form part of some engineering investigation intended to determine whether or not a certain material is suitable for a specific purpose. Almost all the existing data relate to the simplest standard creep test for constant load. The results are presented in the form of so-called creep curves.

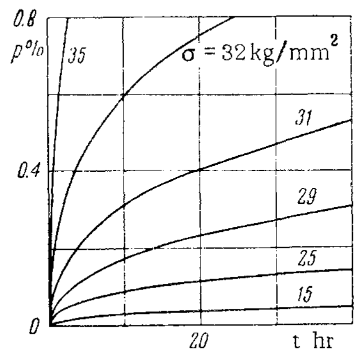

Fig. 2.

Some typical creep curves are shown in Figs. 1, 2, and 3. Figure 1 gives the results of a unique series of tests on carbon steel at 450° lasting 100 000 hours (12 years) [1]. It illustrates some typical features of creep curves, which it is customary to divide into three parts: a primary stage of nonsteady creep up to 5000 hours, a secondary stage of steady-state creep, during which the creep rate is constant, and a tertiary stage of accelerated creep preceding fracture. In the two lower curves the secondary stage is well expressed and no transition to the tertiary stage is observed. In the upper curves, corresponding to a higher stress level, there is only a nominal secondary stage, since a constant creep rate is, strictly speaking, not recorded.

Figure 2 gives curves for tests of medium duration on an aluminum alloy (D-16AT at 150°) [2]. In order to get an appreciable deformation in a short time, it is necessary to raise substantially either the stress level or the temperature. Clearly, most of the deformation occurs in the primary stage, the secondary stage being hard to distinguish: at high stresses the first stage goes directly over into the third, at low stresses the creep rate visibly continues to decrease throughout the test.

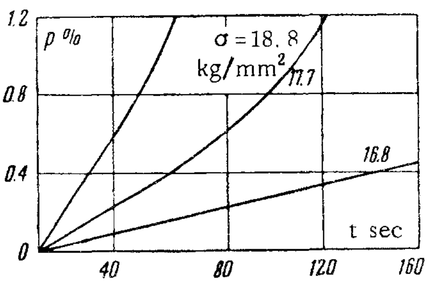

Fig. 3.

Finally, Fig. 3 shows the curves for a short-time creep test on alloy D-16AT at the very high (for the material) temperature of 250° (S. T. Mileiko). In phenomenological theories of creep the creep curve is regarded as the primary experimental fact. If every component worked only in simple tension at constant load and temperature, a theory of creep would not be necessary; all the calculations could be based on creep curves. In fact, a series of creep curves for different stresses and temperatures is a graphic means of representing the relation

$$e = f(\sigma, T, t) \tag{1.1}$$

where e is the strain, σ the stress, T the temperature, and t time. Obviously, different methods of defining the functional relationship are completely equivalent; the tendency to represent this dependence with the aid of a certain set of elementary functions is solely a question of convenience, not of principle. Therefore, as our primary fact we shall assume the existence of relation (1.1) as such, and regard all analytic representations of the law of creep as approximations of varying degrees of convenience and accuracy.

This point deserves special emphasis. The outcome of a physical theory is usually some formula obtained as a result of definite assumptions concerning the micro-mechanism of the process. More or less approximate correspondence between this formula and the experimental data indicates that the assumed mechanism does in fact predominate (at least this is how experimental results are usually interpreted). For the designer concerned with the life of a component made of a given material such a formula is only a compact means of designating the properties of the material; he will use a theoretical formula, if it is accurate enough, but prefers an empirical one, if it describes these properties more exactly. The engineer is interested not in the form of the functional relationship, but in the behavior of those parameters on which a given quantity depends, and in establishing the approximate limits within which certain very simple hypotheses that limit the number of these parameters hold true.

Let us return to Eq. (1.1). It is not the expression of any physical law, since it only describes the results of an experiment set up under perfectly definite, narrow conditions, when σ = const and T = const. If σ and T are functions of time, then, generally speaking, predictions based on Eq. (1.1) will be unreliable.

By a phenomenological or mechanical law of creep (for uniaxial tension) we mean some relation between the functions e(t), σ(t), and T(t) containing certain time operators – differential, integral, or otherwise. When σ = const and T = const, this equation must yield Eq. (1.1), but it must also correctly describe other possible cases, when σ or T varies in an arbitrary manner with time. In reality, the experimental possibilities are limited, so that we are always left to make the best possible approximation to some set of experimental data.

We shall introduce into the starting relation a large number of operators, constants, and functions; let us introduce, say, derivatives up to a certain sufficiently high order. Clearly, if the number of constants is equal to the number of experimental points, then any theory will be confirmed. The problem is to get away from this path, to impose rational limitations on the excessive possibilities of representation, and to construct a simple phenomenological scheme that still takes into account the principal features of the effect.

In this case the requirement of simplicity means that it should be possible to use the theory in practical engineering calculations and to obtain the final results with a certain reasonable, but not extreme, degree of accuracy. It is then essential to determine the structure of the phenomenological equations, to know whether they can be formulated as relations between directly measurable quantities or must contain certain structural parameters governed, in their turn, by definite kinetic equations, and which of these parameters is characteristic for a certain material or set of working conditions. The concrete form of the functional relationships is not particularly important: if they are derived from a physical theory, it is possible to reduce the number of experiments required. In most cases, mechanical theories use experimental data obtained directly from macro-experiments and presented either in the form of graphs or with the aid of empirical formulas. The latter are useful for interpolating experimental data, whereas the question of the laws of application of these formulas for purposes of extrapolation goes beyond the framework of mechanics.

We shall note certain features of the problems of the engineering theory of creep, with which it is necessary to reckon when comparing the data of physical and engineering theories.

1) The designer is usually interested only in very small deformations. The problem of designing for creep is a double one. On the one hand, there must be no accumulation of residual strains leading to interference with the proper functioning of the part. Ordinarily, the permissible strain is very small, for example, 1% of the total deformation due to increase in the diameter of a turbine wheel during operation. Often the requirements are even more strict – local deformation not to exceed 1%. Hence it is necessary to determine the so-called nominal creep limit, the greatest stress at which the creep strain during a given time, the nominal working life, does not exceed 1%. On the other hand, creep leads to failure, the strain at the moment of rupture decreasing with increase in the time of application of the load and hence with decrease in stress. Under actual working conditions modern heat-resistant alloys fail at very small strains, often less than 1%.

2) Depending on the nature of the component, the creep process may last from several seconds to several years or decades, the permissible strains being of the same order; thus, we have to deal with completely different temperature and stress levels. The region of investigation of creep may conventionally be divided into the following parts:

a) Long-time creep — months and years. Chief area of application — stationary steam and gas turbines. Object of investigation — primarily steels of the ferritic and austenitic class. Creep is complicated by phase and structural transformations, the behavior of the materials is individual, the phenomenological laws are rather rough and approximate. The creep is usually considered steady-state, since the design regimes are stationary.

b) Medium creep — hours and days. This kind of creep, at a higher level of stresses and temperatures, is encountered in gas turbine power plants, notably in aircraft. The problem is complicated by the fact that such components generally operate at variable loads and temperatures; the primary phase of creep is very important.

c) Short-time creep — seconds and minutes. Interest in short-time creep has been aroused only quite recently. At a very high temperature level for a given material creep is essentially a process of quasi-viscous flow, and the material can be likened to a liquid with nonlinear viscosity.

Thus, the range of times during which roughly the same strain may be registered is about eight orders of magnitude, but components of a given type operate within a comparatively narrow range; therefore it is preferable to speak not of the theory of creep but of theories of creep, each covering a certain section of this range and varying with the properties of the material and the practical problems it is required to solve.

Every investigator, when he first runs a creep test, is struck by the wide scatter of the experimental data and the extreme instability of the creep characteristics, which vary from specimen to specimen — the apparent irreproducibility of the experiment.

For this reason it is possible to state the following. The job of a metallurgist designing new alloys for use at high temperatures is to block creep processes, thus ensuring the stability of the material under working conditions. Hence, its characteristics will also be more stable. In this sense, modern heat-resistant alloys are a more suitable material for investigation than, say, carbon steels or D-16 aluminum alloys. On the other hand, the dependence of the creep rate on stress and temperature is very strong. Therefore the result should not be considered a poor one if the creep strain in two apparently identical experiments is found to differ by, say, 50%.

Let us phrase the question differently and ask what stress it is necessary to apply to a specimen to obtain a given strain rate. It appears that the difference in stresses will be of the order of 10%. This answer suits the designer perfectly, since he is concerned with permissible stresses or permissible loads; the difference is covered by introducing a certain safety factor.

As an illustration of this approach, we may cite the experiments of TsNIITMASh (Central Scientific Research Institute for Heavy-Duty Machines) [3,4] involving destructive creep tests on turbine discs designed in accordance with a certain theory. The time to rupture varied within quite wide limits. If the theoretical times to rupture are compared with the actual times, the conclusion may not be favorable to the theory, but an evaluation based on stresses shows that the difference between theory and experiment does not exceed 6%, or 3% with respect to the nominal disc speed. This is a satisfactory result as far as practical requirements are concerned.

2. Steady-state and quasi-steady-state creep. Over the linear part of the creep curve the creep rate does not depend on time but only on the stress and temperature. This may be written as follows:

$$\dot{e} = v(\sigma, T). \tag{2.1}$$

To obtain a rough estimate of the cumulative strain to a first approximation we may take

$$e = v(\sigma, T)\, t.$$

Here we neglect not only the strain of the primary stage but also the elastic, and possibly the instantaneous plastic deformation.

Equation (2.1) may be treated as a law of creep, i.e., regarded as suitable for variable as well as constant stresses and temperatures. This means that the material resembles a liquid with nonlinear viscosity; Eq. (2.1) is the most general law of viscous flow. In actual fact, we must distinguish two cases.

a) Steady-state creep follows the primary stage. It appears that each time the load changes a new primary stage begins. As an illustration, consider Fig. 1. For technical reasons the test was interrupted after 37 000 hr and resumed at the same stress some time later. A new, fairly long primary stage was observed and only after a certain time was a constant creep rate (the same as before the interruption) restored. For long-time creep Eq. (2.1) does not express a physical law; it can be used only conditionally, with a definite (rather low) degree of accuracy.

b) Short-time creep. In Fig. 3 the stage of constant creep has an entirely different character. Whereas in the first case the change in creep rate in the primary stage denoted a change in the properties of the material, while transition to the secondary stage denoted a certain stabilization of these properties, now the creep rate is invariant from the very beginning and there is no change in the properties of the material during the creep process (disregarding, for the time being, the question of a tertiary stage). In fact, with short-time creep in most materials the creep rate is uniquely determined by the instantaneous values of the stress and temperature and is completely independent of the past history.

A somewhat more far-reaching generalization of the hypothesis of steady-state creep consists in the following. Suppose we change the time scale, defining a certain function $\tau(t)$ so that in the coordinates e-τ the creep curve becomes a straight line. Then, if we define the velocity in terms of the modified time τ, Eq. (2.1) will describe the entire creep curve, whatever its shape.

Of course, relation (2.1) still can not be regarded as a physical law. It may be understood in the sense that the structure of the material undergoes certain changes with time, the kinetics of which are completely independent of the applied stress.

This view has been advanced before and is known as the aging hypothesis. It is linked with certain quite important advantages of a computational kind. If the load varies slowly, predictions based on this hypothesis are not bad.

If the material behaves in a complex way, if it suffers phase transformations and the characteristics of the creep curves are such that they can not be interpreted by a more rational method, possibly the simplest and most reasonable way of designing real components is to adopt the hypothesis of aging and use the primary creep curves as they are obtained.

An even simpler method is to base the calculations on (1.1). The experimental creep curves are replotted in the form of so-called isochronous or stress-total strain isocurves, each curve corresponding to a given value of time. The creep calculations reduce to a series of calculations based on the theory of plastic deformation with the plastic stress-strain diagram replaced by the corresponding isocurve. This relatively simple and rough method is now very widely used and gives reasonable results under stationary conditions.

By processing a large amount of experimental data a number of authors have established that the isocurves may be considered similar; thus,

$$\sigma = \varphi(e)\,\vartheta(t).$$

It has been found that this relation is well approximated by the expression

$$\sigma = \frac{\varphi(e)}{1 + bt^{\beta}}, \tag{2.2}$$

where $\sigma = \varphi(e)$ is the equation of the instantaneous tension curve; the value of β for metals is fairly constant, viz. $\beta \approx 0.3$. Formula (2.2) was thoroughly checked in [5, 6] and recommended (with certain reservations) for the extrapolation of creep data over long periods.

Returning to steady-state creep, we note the commonest approximations of the dependence of $\dot{e}$ on stress. For constant temperature

$$e^{\cdot} = \varepsilon_n \left(\frac{\sigma}{\sigma_n}\right)^n, \qquad e^{\cdot} = 2\varepsilon_e \operatorname{sh}\frac{\sigma}{\sigma_e}, \qquad e^{\cdot} = \varepsilon_e \exp\frac{\sigma}{\sigma_e} \tag{2.3}$$

where ε_n and ε_e, σ_n and σ_e are constants. The first of Eqs. (2.3), where two dimensional constants are introduced for convenience, is the most widely used. The hyperbolic sine laws is obtained naturally in a number of physical theories of creep, so there is now a tendency to give it preference. The third equation practically coincides with the second, if the stresses are not too small, and is more convenient to use. However, it should be borne in mind that it is not applicable at small values of σ/σ_e.

The first of Eqs. (2.3) may be written more correctly as follows:

$$e^{\cdot} = \varepsilon_n \left|\frac{\sigma}{\sigma_n}\right|^{n-1} \frac{\sigma}{\sigma_n}. \tag{2.4}$$

In this case we assume that the creep rates in tension and compression are the same, if the absolute values of the stresses are the same. This is to all intents and purposes confirmed by experiment (cf. [7]).

The power law of creep has certain practical advantages. It implies that if a body is exposed once to external forces P_1 and a second time to forces P_2, proportional to the former, and if the stresses and velocities of points on the body are, in the first case, σ_1 and u_1 and, in the second, σ_2 and u_2, then

$$\sigma_1 : \sigma_2 = P_1 : P_2, \qquad u_1 : u_2 = P_1^n : P_2^n \, . \tag{2.5}$$

On the other hand, the second and third of Eqs. (2.3) and Eq. (2.5) can be written in dimensionless form, without any of the material constants; therefore the theoretical solutions obtained are universal in character.

In analyzing the data of our experiments, we have used the exponential law, but we can not claim that it has any decisive advantages over the power law. It is impossible to describe the creep process over the entire range of stresses with the same constants. Thus, in [2] it was shown that in using the exponential law it is necessary to take one set of values of the constants in the region of low stresses and another set for high stresses, when the instantaneous strain includes a plastic component. The same applies to the power law.

Moreover, on analyzing certain experiments relating to the complex stress state we see that the transition from a low to a high stress level is associated with certain qualitative changes in the creep process. In this case analysis based on the power law enables the transition point to be determined more clearly.

Note that the purely formal, analytical advantages of the various forms of approximation play an important part in the mechanical theory of creep. It is often necessary to use simple relations obtained as a result of averaging a large number of scattered experimental points rather than more complex formulas, which may be hard to apply even to the analysis of experimental data, let alone the solution of theoretical problems.

Considerably more complicated is the question of the temperature dependence of the creep rate. The simplest hypothesis reduces to the fact that the constants ε_n and ε_e are functions of temperature

$$\varepsilon_n = \varepsilon_n^\circ \psi (T) . \tag{2.6}$$

For us it is not too important that

$$\psi (T) = \exp\left(-\frac{U}{kT}\right) . \tag{2.7}$$

It appears that the activation energy is not constant. A study of its temperature dependence throws light on the micromechanisms of creep and has physical importance. For computational purposes it is more convenient to approximate the function ψ (T) with some suitable, sufficiently simple expression.

On the other hand, data on the primary stages of creep have been analyzed with the aid of the equation

$$e = c \exp\left(\frac{\sigma}{\sigma_e}\right) t^m$$

and it has been observed that all three constants, i.e., c, σ_e, and m, depend on the temperature. The temperature dependence of m means that the shape of the creep curve itself varies with temperature. Only in a limited temperature range, of the order of 50° for steels, can σ_e and m be regarded as constant, whereas for c relation (2.7) is assumed. This applies to a fairly wide range of materials including copper, low-alloy steels, austenitic steels, and aluminum alloys.

The question of a more accurate description of the temperature dependence of creep is important in another respect, which, strictly speaking, is not related to the problem of the mechanics of creep, but nonetheless is closely linked with it.

Often in designing a part we have an insufficient supply of experimental creep data. In particular, this is true of parts intended to remain in service for a long time. It is therefore necessary to resort to extrapolation of the data for short-time tests. This question is of enormous practical importance and still unsolved. Evidently, the essential shortcoming of many such attempts is that they are concerned with creep in general; efforts are directed toward finding universal criteria, whereas, in actuality, materials and working conditions are more individualized. It is therefore important to establish principles that will enable us to distinguish certain individual groups of materials with essentially different rules of extrapolation. It is worth-while reporting certain points bearing on this problem, though the literature on the subject is extremely voluminous and the problem itself is closely interwoven with the physics of creep. The same extrapolation formulas are generally used both for creep and for failure. The question of failure will be considered later. For the time being it may be noted that most authors assume, explicitly or otherwise, that failure corresponds to a certain strain accumulated in the secondary stage of creep. This critical strain e* is independent of both stress and temperature [9, 10].

This view, which is based on the notion of a single mechanism governing both the creep rate and the failure rate, can not pretend to universal significance, if only because, in general, creep curves often do not have a secondary stage. Moreover, the total strain at rupture by no means remains constant; as a rule, it decreases with decrease in the rupture stress for a given temperature.

We shall take the relation between creep rate, stress and temperature in the form:

$$e^{\cdot} = \exp\left(-\frac{U(\sigma)}{kT}\right)\text{const}. \qquad (2.8)$$

In the special case when $U = U_0 - \gamma\sigma$, where γ is a structure-sensitive constant, we get Zhukov's formula [9, 11]. If the rupture life is t^*, then $t^* = e^*/e$, and from (2.8) we get

$$\tau_1 = T(c + \log t^*) = F(\sigma). \qquad (2.9)$$

The quantity τ_1 is called the Larson-Miller parameter [12]. The single constant c is fairly easy to determine. The points corresponding to rupture for different times and temperatures must lie on a single curve, if along one axis we plot values of the parameter and along the other the stress. From the origin of the Larson-Miller parameter it is clear that it can be used not only for the extrapolation of data relating to long-time strength but also in relation to creep, if e^* is understood to stand for some fixed strain.

Another widely used method of extrapolating long-time strength data is that of Manson and Haferd [13], who introduced, in a purely empirical way, the parameter τ_2

$$\tau_2 = \frac{T - T_a}{\log t^* - \log t_a} = F(\sigma) \qquad (T_a = \text{const},\ t_a = \text{const}). \qquad (2.10)$$

Many authors have tried to verify the possibility of predicting data on creep and long-time strength using the above criteria and others, with our without some theoretical basis. In general, their conclusions are rather indefinite. According to the majority of experimenters [14, 15], the best results are given by the Manson-Haferd formula, though any of the formulas may be used for a first rough approximation.

$T°$	σ, kg/mm²	t^*, hr	Theoretical rupture stress for actual life according to formulas (2.9)-(2.11) and percentage error					
			2.9	%	2.10	%	2.11	%
				Nimonic-90				
650	4090	4110	4090	0	4090	0	4160	+2
	3780	6440	3900	+3	3840	+2	3940	+4
	3620	5370	3960	+9	3940	+9	4060	+12
	3150	15290	3540	+23	3440	+9	3630	+15
700	2040	12090	2440	+19	2300	+12	2470	+21
	1970	20400	2230	+14	2040	+4	2230	+14
750	1570	7590	1570	0	1500	−5	1600	+2
	1100	22160	1260	+14	1040	−6	1210	+10
815	865	3340	895	+4	820	−5	880	+2
870	315	16292	284	−10	220	−30	236	−25
				Nimonic-80				
650	3150	5270	3210	+2	3100	−1.5	3220	+1.5
	2830	8170	2990	+6	2850	+0.5	2980	+5
	2520	13390	2760	+10	2520	0	2770	+10
700	2040	4840	2140	+5	2010	−1.5	2180	+7
	1570	10900	1820	+16	1590	+1	1820	+16
	1100	34060	1390	+26	1130	+3	1370	+24
750	1260	4450	1260	0	1240	−1	1240	−1
	945	13090	975	+3	880	−7	945	0
	630	22660	835	+33	725	+15	790	+25

By way of illustration, the table contains data on the analysis of long-time (up to 34 000 hr) tests on the alloys nimonic-80 and nimonic-90 [16]. It gives the stress errors in the prediction of rupture life using the two criteria mentioned and a third derived from (2.7) that gives an estimate of the life based on the modified time

$$\tau_0 = \int_0^t \psi(T)\,dt \qquad (2.11)$$

It is worthwhile dwelling in more detail on [17]; the data it contains very clearly illustrate the difficulties involved in using different criteria. Figure 4 gives the stress as a function of the Larson-Miller parameter for an austenitic steel. For each temperature the points lie on a particular straight line. However, if for all the temperatures we draw through all the points a common straight line, then none of the points will lie very far away from it. By joining the experimental points for a single temperature and producing the straight line obtained, we get a seriously erroneous result.

Thus, with the aid of the Larson-Miller parameter we can interpolate, but not extrapolate. The positive side of [17] may be summarized as follows. A general relation between rupture life, stress and temperature is proposed:

$$\log t^* = a \log \sigma + b(T).$$

Depending on the nature of the quantity a, the material may be divided into four groups:

$$(1)\quad a = \text{const}, \qquad (2)\quad a = a(\sigma), \qquad (3)\quad a = a(T), \qquad (4)\quad a = a(T, \sigma).$$

For the first three groups it is possible to find certain relatively simple methods of presenting the results. For instance, the data on austenitic steel, presented in Fig. 4, were correlated by making a suitable choice of the time-temperature parameter τ; these results are given in Fig. 5, where all the points already lie on a single straight line. For the third group the Manson-Haferd criterion holds. The most complex behavior is that displayed by the fourth group, which includes, for example, the chrome-nickel-cobalt alloy S-590 investigated in [18].

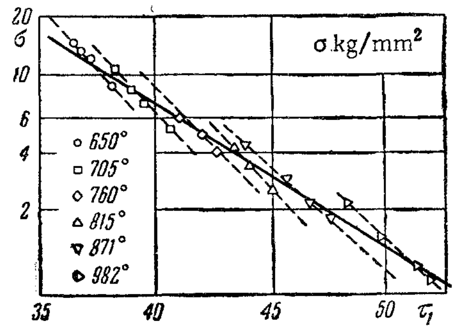

Fig. 4.

Unfortunately, the author does not suggest how material might be related to one of the four groups in advance, when complete test data are not available. Thus, the study is purely empirical in nature and does not advance the solution of the problem of the extrapolation of creep data. Nonetheless, the analysis is very instructive. It gives a good idea of the difficulties involved and shows how carefully the various theoretical formulas must be related to actual material of a complex type. Each theoretical formula presupposes a certain perfectly definite deformation mechanism, whereas in engineering alloys different mechanisms may exist side by side, and pure examples can be isolated only under special laboratory conditions of an artificial kind.

Nonsteady creep. Creep processes of medium duration have become important since the gas turbine was introduced as an aircraft power plant, and the aircraft designer began to be concerned with the heating of wings and fuselage. Essentially, these processes relate to the first part of the creep curve, and the study of this part of the curve occupies an independent place in the theory. In the preceding section, we draw attention to the possibility of a formal treatment of nonsteady creep in the same terms as steady-state creep. Of course, such a treatment could not be completely satisfactory, so we are obliged to seek other solutions. Without going into a detailed analysis of the various possibilities, let us consider two of the most fruitful concepts.

a) Creep as a memory process. Boltzmann's old and well-known idea [19], later developed by Volterra and others [20], led to the creation of the so-called elastic memory theory of creep. In this theory Hooke's law is replaced by a relation of the form:

$$Ee(t) = \sigma(t) + \int_{-\infty}^{t} K(t-\sigma)\,\sigma(\tau)\,d\tau.$$

The memory theory can be generalized in various ways to include the case of creep of metals when the relation between stress and strain is essentially nonlinear. The present author has proposed the equation:

$$\varphi(e) = \sigma + \int_{-\infty}^{t} K(t-\tau)\,\sigma(\tau)\,d\tau \quad \text{for} \quad e^{\cdot} > 0 \tag{3.1}$$

as well as an equation with the left side linear in e for relaxation. Other variants of the nonlinear memory theory were developed in [22-24]. In formulating his hypothesis the author makes use of two basic facts.

1. The similarity of the isocurves as expressed by (2.2). The function φ (e) figuring in (2.2) and (3.1) determines the instantaneous strain curve; formula (2.2) is obtained from (3.1) by choosing a suitable kernel.

2. Creep recovery. Experiments [25] show that recovery is a linear process of the elastic aftereffect type. Equation (3.1) gives a qualitatively correct description of this phenomenon.

In practice, the memory theory is rather difficult to apply. A more detailed investigation [26] has shown that in reality the role of recovery is less important than predicted by Eq. (3.1). At the same time, the memory theory has found wide application in connection with concrete [23], rocks [27], plastics, and polymers [28, 29], where the nonlinearity is more weakly expressed and in part need be taken into account only as a correction applied to the basic result.

b) Equation of state or hardening hypothesis. It is natural to regard the creep rate as being uniquely determined as a function of stress and temperature for a given structural state. We shall assume that the structural state can be defined with the aid of a finite number of parameters of state. This means that if two specimens have the same numerical values of the parameters of state, it is impossible to establish differences between these specimens with respect to structural properties of any kind.

In accordance with the above assumption, at a given temperature the creep rate due to the same stress will also be the same for the two specimens. The number of structural parameters may be very large, although this number may not be confused with the number of structural properties by which states are identified. It is also impossible to maintain that all the structural parameters play a part in the creep process. A material with a different structure may give the same creep rate under the same conditions. Thus, we shall assume that the creep rate is a function of the stress, the temperature, and a certain set of structural parameters q_i

$$p^{\cdot} = v(\sigma, T, q_i) \qquad (i = 1, \ldots, n) \tag{3.2}$$

In their turn, the quantities q_i vary during the creep process in accordance with certain kinetic equations. In order to obtain the complete system of equations of creep, it is necessary to link these kinetic equations with Eq. (3.2).

The following assumption will be general enough:

$$dq_i = a_i dp + b_i d\sigma + c_i dT + d_i dt \tag{3.3}$$

where p is the creep strain, while the quantities a_i, b_i, c_i, d_i may depend on p, σ, T, t, q_j. The simplest assumption is that the only structural parameter is the hardening parameter, which is uniquely linked with the cumulative creep strain. As this parameter we can take the quantity p. We then get the equation usually known as the equation of state for creep:

$$p^{\cdot} = v(\sigma, p, T) \tag{3.4}$$

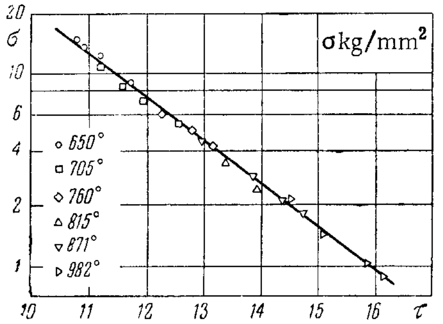

Fig. 5.

In this equation p represents the creep strain, but not the total plastic strain. It is known [30] that a small preliminary deformation has no effect on the creep curve. In [31] the author tested this fact more thoroughly and established that an instantaneous plastic strain of the order of 1% has no appreciable hardening effect. In general, however, the very complex interaction between the instantaneous plastic strain and the creep strain has received little attention.

In [32], where an investigation was made of the effect of preliminary plastic deformation on the creep of copper, it was established that large plastic strains affect creep appreciably, in a rather remarkable way. For the region of small plastic strains, in which the designer is primarily interested, the above conclusion is confirmed. In any case, the introduction into (3.4) of the total plastic strain instead of the creep strain leads to a serious error, whereas neglecting the hardening role of the instantaneous deformation is an acceptable approximation as far as engineering calculations are concerned. On the other hand, preliminary creep has been observed to have an appreciable effect on the instantaneous strain curve [33].

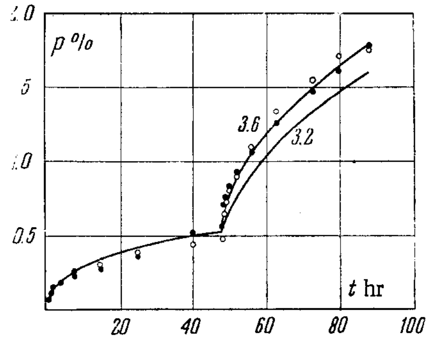

Fig. 6.

Experimental verifications of the kinetic equations of creep have mainly been along the following lines.

a) Comparison of creep and relaxation data. In relaxation tests the elongation of the specimen is kept constant while the fall in stress is measured. Putting $\dot{e} = 0$, we can integrate (3.4) and find the stress as a function of time. There is a known graphic method of constructing relaxation curves from creep curves [34]. We preferred to take an approximate expression for the creep curves and assign suitable values to Eq. (3.4), which is integrated for the relaxation case. The results of predicting relaxation curves from creep curves based on the very simple hypothesis expressed in Eq. (3.4) are completely satisfactory.

b) Creep under stepwise loading. The case of creep with a sudden stepwise change in load gives greater contrast than the case of relaxation, when the stress changes smoothly. With this method of testing the errors of the predictions based on different variants of the theory are more clearly expressed. At the same time, the simplest hardening hypothesis (3.4) is verified indirectly without any calculations involving approximations. Similar tests have been carried out by the author of this paper and by a group of Japanese authors. The work of the latter is reviewed in [35]. In the form (3.4) the hardening hypothesis is valid only as a first approximation. Experiments show a systematic deviation from the results predicted by the theory. The nature of this deviation is as follows.

1. With increase in load the creep rate grows considerably more rapidly than follows from the theory. The theoretical value is approached only after a certain time has elapsed. The total strain is somewhat greater than the theoretical.

2. With decrease in load the creep experiences a certain "pause," then it gradually increases and attains the theoretical value.

These facts have been verified for quite different materials both by the present author [33, 2, 31, 36] and by the Japanese [35].

c) Creep with small changes in the stress state. In the theory of stability of structural elements at high temperatures Eq. (3.4) is varied. For small increments in the stresses and strains relative to the stresses and strains of the basic state we get a certain rheological relation recalling the known Thomson equation but with variable coefficients. In [37] data are given on the direct determination of the coefficients of this equation. They differ quite sharply from those obtained by varying (3.4).

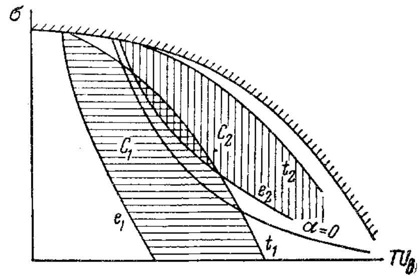

Fig. 7.

In the above-mentioned experiments the problem was essentially to find the conditions for which the contradictions between theory and experiment are most clearly expressed. In practice, the variation in load is usually quite small and the simplest theory gives perfectly satisfactory results —as in the relaxation problem. The theory is also applicable to the case of temperatures varying slowly within narrow limits. Although in this case we get effects similar in character to those associated with variable stresses, the theory permits fairly reliable calculations, even for temperatures that very cyclically.

Let us now return to the more general relation (3.2), (3.3). In [35] the authors deal essentially with the case where n = 1 and

$$dq_1 = a\ dp + d\ dt. \tag{3.5}$$

No definite hypotheses were proposed in connection with a and d and a number of details are missing from the publications; however, the authors state that by obtaining these coefficients experimentally they can get a very good description of creep under variable loads. Relation (3.5) may be regarded as a new formulation of the old idea of interaction between hardening and softening in the creep process; the second term expresses softening if d < 0, aging if d > 0.

Let us consider several other possibilities of introducing structural or hardening parameters. The simplest assumption

$$dq_1 = \sigma\ dp \tag{3.6}$$

implies that the energy dissipated in creep is taken as a measure of hardening. It turns out that this minor modification of the general hardening hypothesis results in an appreciable improvement in the degree of correspondence of theory and experiment [38]. Figure 6 shows experimental points for the creep of alloy D-16AT at 150° for a stepwise varying load and curves calculated from Eqs. (3.2) and (3.3) with hardening parameter (3.6).

In [39] there was introduced yet another structural parameter q_2 such that

$$dq_2 = p\ d\sigma. \tag{3.7}$$

With stepwise variation in load this parameter receives a finite increment. It gives quite a good description of several of the effects noted.

Thus, we conclude that with an accuracy sufficient for most practical purposes the creep of structurally stable materials under variable loads may be described with the aid of the simple equation (3.2). At the expense of a slight refinement of the basic hypothesis the accuracy can be much improved. True, these results relate to the case in which the initial deformation is elastic. In the plastic region, the picture is somewhat more complicated, but the general approach described offers a real means of seeking the corresponding equations.

For practical purposes it is convenient to put the law of hardening in a certain acceptable analytic form, to which there is no need to assign an absolute value. It is simply a matter of a more or less convenient approximation of the experimental data. For many materials the first sections of the creep curves are approximately the same. Equation (3.2), for example, may be written in the form:

$$\dot{p} = P(p)\,S(\sigma). \tag{3.8}$$

For the time being, let us leave the question of temperature dependence on one side. Numerous authors, starting with Andrade [40], have established that over the first section of the curve the creep strain is proportional to t^m, where $0 < m < 1$. Andrade obtained a value of 1/3 for m and regarded this as a physical law. In reality, m may assume various values and for the same material is dependent on the temperature. Now the function P (p) is determined uniquely, and the law of hardening assumes the form

$$\dot{p} = p^{-\alpha} S(\sigma), \qquad \alpha = \frac{1-m}{m}. \tag{3.9}$$

The function S may be either a power function or an exponential function, like v in the law of steady-state creep. Certain limitations affecting the form of the function S (σ) were noted in [41]. In a first approximation the temperature dependence can be taken into account with the aid of the factor ψ (T) on the right side, although this, too, is not exact. It has already been pointed out that the exponent m also depends on the temperature. We still lack more accurate information about the temperature dependence of the law of hardening.

Let us assume that the strain is proportional to t^m and that at the initial moment the creep rate is infinitely large. Actually, it is impossible to determine the dividing line between the end of instantaneous deformation and the beginning of creep deformation in a creep test, and measurements of the instantaneous deformation are inaccurate. Attempts to determine the initial creep rate have been unsuccessful. The behavior of the material in the first fractions of a second following application of the load depends appreciably on the method of loading, as determined by the dynamic characteristics of the testing machine. Therefore, in a number of cases we used the following technique. The load was applied not instantaneously, but in accordance with a quite definite law. Usually a constant increase in stress was ensured by means of a hydraulic or electromechanical device. From the very beginning the elastic and plastic deformation is accompanied by creep. By assigning a definite form to the creep law, it is possible to reconstruct theoretically a hypothetical instantaneous deformation curve.

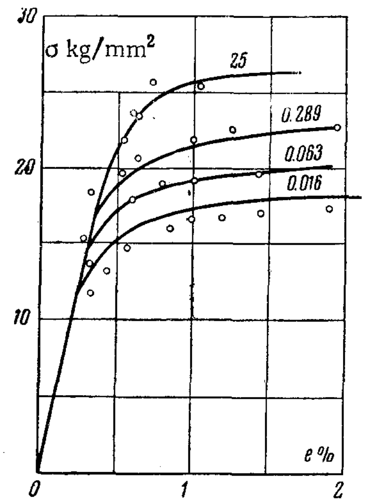

Fig. 8.

The problem of the transition from nonsteady to steady-state creep may be solved in more than one way. One approach presupposes that with increase in p the function P (p) in (3.8) tends to a finite limit, which is reached either for a finite value of p or as $p \to \infty$. One possible calculating scheme based on this principle is described in [42].

A second approach is also possible. It is assumed that nonsteady creep and steady-state creep are controlled by different mechanisms and coexist simultaneously and independently; then the total strain at any moment consists of the instantaneous elastic or elastoplastic deformation e_0, the deformation of nonsteady creep p, and the deformation of steady-state creep e_1, which proceeds continuously at a constant rate. The nonsteady component is described by a certain equation of the same type as the equation of state (3.4), but damps out with time, the creep rate approaching zero.

The investigation of nonsteady problems in accordance with these two principles is carried out by quite different methods. Considerations of simplicity and convenience compel us for the moment to prefer the first principle, although we still lack the experimental data for making a really rational choice.

It is possible to go still further in analyzing the mechanism of creep. Thus, being anxious to take recovery into account, the author of [43] isolated the component corresponding to the nonlinear memory type of strain. Analysis of the experimental data for celluloid, in which recovery is very important, gave encouraging results.

In recent years a great deal of attention has been paid to short-time creep lasting seconds and minutes. A high level of stresses and temperatures creates specific conditions; for the majority of materials creep is not accompanied by hardening and takes the form of a quasi-viscous flow superimposed on the ordinary elastoplastic deformation. If we attempt to analyze tests run at different loads and temperatures with the aid of Eq. (3.9), we find that with increase in load and temperature the exponent α tends to zero.

Before considering the experimental data on short-time creep in more detail, let us examine the choice of limiting working conditions for parts operating at high temperatures. We shall plot temperature and stress along the coordinate axes, as shown in Fig. 7. The upper curve corresponds to instantaneous rupture; the region of working regimes lies wholly below this curve. If the part is designed for a lifetime t_1, we can plot a long-time rupture curve for t_1. On the other hand, if the creep strain is small, it can generally be neglected in the calculations. Assigning some reasonable tolerance, we can plot a curve e_1 corresponding to the accumulation of strain at the limit of this tolerance in the time t_1. The shaded region C_1, bounded by the curves t_1 and e_1, is the region in which it is necessary to have the creep characteristics in order to design a part of the given type. Now if the part is designed for a shorter lifetime t_2, the corresponding curves t_2 and e_2 will lie above the curves t_1 and e_1, and the corresponding region of working regimes will be C_2. On the same plane we can plot the curve corresponding to the boundary of the region for which $\alpha = 0$; for sufficiently small t_2 the region C_2 lies wholly above the curve $\alpha = 0$. Short-time creep investigations in our laboratory were based on the following hypotheses.

1. Each temperature has its own curve of instantaneous deformation.

2. The creep rates depend only on stress and temperature and not on the past history, particularly the previous plastic strain. These assumptions were verified in various ways with perfectly satisfactory results. Figure 8 shows stress-strain curves (alloy D-16AT at 250° for a constant loading rate; the figures on the curves denote the rate of change of stress in kg/mm^2 per second) for different rates of change of load (difference equivalent to four orders) calculated in accordance with this hypothesis. The experimental points are also shown. The instantaneous curve is not very different from the curve recorded for maximum rate of change. The creep law was determined independently – from tests at constant stress. It should be noted that we tested engineering alloy as supplied by the manufacturer; accordingly, the individual scatter is fairly high. The theoretical curves were constructed from averaged characteristics and compared with the data for a single specimen. The difference does not exceed the limits of scatter.

Another method of confirming that the creep rate does not depend on the past history is to run a repeated relaxation test, when the relaxed specimen is returned to the starting stress, relaxed again, and the process repeated several times. All the repeated relaxation curves coincided perfectly, which indicates absence of hardening.

The results of short-time creep tests on aluminum and titanium alloys and various types of steel, partially published by S. T. Mileiko in [44], enabled the present author to propose the following empirical formulas for the creep rate as a function of stress and temperature:

$$\begin{aligned} \dot{p} &= k \exp(\alpha\sigma + \nu T) \qquad \text{for} \quad t < T_1, \\ \dot{p} &= k \exp\left(\frac{\alpha_0}{T_0 - T} + \nu T\right) \quad \text{for} \quad t > T_1. \end{aligned} \tag{3.10}$$

Here α, α_0, ν, T_0, T_1 are constants. In the transition region, when $T \approx T_1$ (275-300° for alloy D-16AT) the scatter of the data is somewhat greater than at other temperatures. However, we shall not dwell on the question of what happens at the boundary between the two forms of (3.10).

4. Failure due to creep. It is only very recently that attempts have been made to include the problem of failure due to creep within the framework of the theory of creep. The simplest scheme is that of viscous fracture [45]. For simplicity, let us assume that the creep is steady-state and that (2.1) also holds true in the region of finite strains. Now the state of deformation must be understood to mean the ratio of the rate of elongation to the instantaneous length of the specimen. Denoting the latter by x, we get $\dot{e} = \dot{x}/x$. It is natural to take the quantity $e = \ln x/x_0$ as a measure of deformation, where x_0 is the initial length. At constant load the stress will increase due to reduction of the cross-sectional area. Assuming that that the volume is invariant, we easily see that $\sigma = \sigma_0 \exp e$, where σ_0 is the nominal stress (i.e., the stress referred to the initial cross-sectional area). We can now integrate the equation

$$\dot{e} = v(\sigma_0 \exp e)$$

It turns out that e attains an infinitely large value in a finite time, which is also taken as the rupture life. For a power law of creep we get

$$\frac{1}{t^*} = n e_n \left(\frac{\sigma_0}{\sigma_n}\right)^n. \tag{4.1}$$

It may appear strange that such a simplified approach sometimes gives good agreement with experiment. One reason for this is that at the end the creep curve is characterized by extremely rapid growth. Another reason is discussed below.

For relatively large times and low stresses we get another kind of failure, characterized by brittle fracture and very small strains. The theory of this kind of failure may be formulated as follows. We shall introduce a certain embrittling parameter ω, which is equal to zero for unstressed material and unity at the moment of rupture. We shall assume that this parameter varies with the stress in accordance with an equation of the type

$$\dot{\omega} = \varphi(\sigma, \omega). \tag{4.2}$$

Integrating this equation, we find the corresponding time to rupture t^{**}, when $\omega = 1$.

In [46] the present author took

$$\dot{\omega} = B\left(\frac{\sigma}{1-\omega}\right)^k,$$

whence he got

$$\frac{1}{t^{**}} = (1+k)B\sigma_0^k. \tag{4.3}$$

An analogous result was obtained in [47], where essentially the same scheme was employed. When $n = k$, the laws of viscous and brittle fracture are exactly the same, which explains the satisfactory results obtained with formula (4.1).

In log-log coordinates the stress-dependence of the rupture life is represented by straight lines in both cases. Usually, the experimental data are closely distributed along one or both of these lines. Between them there exists a certain transition region, which has been investigated by a number of authors.

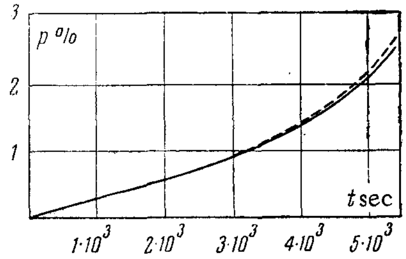

Fig. 9.

It should be borne in mind that rupture is preceded by a tertiary stage of accelerated creep. With pure metals this can apparently in many cases be related to the reduction in cross-sectional area, as follows from the scheme of viscous fracture. This scheme, however, does not take necking into account, i.e., the localization of strain starting from a given moment. At the same time, in many alloys, especially at low stresses, the tertiary stage appears at very small strains of the order of 1-2%. An elementary calculation shows that this can not be attributed to a change in cross-sectional area. We are left to assume that the development of embrittlement, i.e., growth of the parameter ω, exerts an influence on the creep rate. Following the general view, we shall assume that ω is one of the structural parameters determining the creep rate. The hypothetical form of the kinetic equations of creep with embrittlement is assumed to be as follows:

$$\dot{e} = a\sigma^n(1-\omega)^{-q}, \qquad \dot{\omega} = b\sigma^k(1-\omega)^{-r} \tag{4.4}$$

These equations qualitatively describe the creep curve, including the tertiary stage, and also enable us to determine the rupture life. For simplicity, we shall not introduce the hardening parameters; therefore the equations will not describe the primary stages, although in principle it is not difficult to arrange for this. Short-time creep without hardening is an example of the direct applicability of Eqs. (4.4). In analyzing the experimental data on short-time creep we assumed that $n = k = q = r$, which permits a fairly good description of the experimental results, both creep curves and long-time strength curves. In Fig. 9 the experimental creep curve (solid line) and the theoretical curve (dashed line) with a tertiary stage are compared (alloy D-16AT at 250° for $\sigma = 13.6$ kg/mm^2). The creep characteristics were determined from the initial sections of the creep curves; the embrittling charactistics were taken from an analysis of long-time strength curves. With the above mentioned equality of the exponents, Eqs. (4.4) yield the following principle. If $t^*(\sigma)$ is the rupture life for a constant stress σ, then for a variable stress rupture will occur when the equality

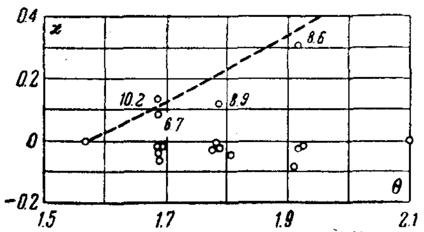

Fig. 10.

$$\int^t \frac{dt}{t^*} = 1 \tag{4.5}$$

is fulfilled.

In the region of short-time creep this principle is fairly well observed, whereas over medium and long periods it is systematically violated, especially when the stress decreases. Evidently, the kinetic equation of embrittlement must have a more complex character; in particular, it is necessary to take into account the possible healing of cracks.

The significance of the phenomenological approach, as expressed by Eqs. (4.4), (4.5) or possibly more complex equations, consists in the following. As a rule, all structures that are designed for creep are statically indeterminate. Therefore the stress distribution is determined by the law of creep and depends on time. The customary method of designing for long-time strength is first to seek the stresses in the elements and then to compare these stresses with the long-time strength curve, if necessary using principle (4.5). In carrying out the first part of the calculation, i.e., in seeking the stress distribution, we do not take into account the opposing effect of incipient failure on the distribution of creep rates, and hence on the stresses. For materials in which embrittlement has little influence on the creep rate and crack formation is local in character, this scheme is applicable. In other cases the formation of cracks distributed more or less uniformly over the volume begins even in the early stages of creep. These join up in the last stage, forming macrocracks, which may be very numerous. This applies, in particular, to areas of stress concentration. In estimating the effect of stress concentrators on strength under conditions of creep, it is necessary to allow for the effect of crack formation on the creep rate, even though only in a rough and hypothetical form.

5. Creep in the complex stress state. All real components, with rare and generally uninteresting exceptions, operate in the complex stress state. At the same time, creep tests on specimens in the complex stress state are technically difficult and the published data are relatively few. In physical investigations this problem is almost always bypassed, and there are no theories, apart from phenomenological ones, of complex-stress creep. The theory of plasticity of metals at normal temperatures, when time effects are absent or negligible, has been better developed than the theory of creep and has, of course, been a model for the latter as regards the construction of different kinds of hypothetical equations.

Any one-dimensional relation between stresses and strains and their rates of change can be nonumerically transformed by converting it into a relation between tensors. In recent years the formal theory of functions of tensors has been intensively developed [48, 49], and the basic difficulty consists in the extremely broad possibilities offered by this theory and the indeterminacy of the criteria for choosing among them. The creep rate tensor p_{ij} must depend on the tensor σ_{ij} and the hardening parameters, which may be scalar quantities or tensors of any rank. If the hardening parameters are scalar, we shall call the hardening isotropic. The hypothesis of isotropic hardening is the very simple hypothesis that forms the basis of the majority of existing theories, although its unsatisfactoriness can easily be demonstrated experimentally [50].

Suppose that we carry out a tensile creep test on a tubular test piece. The creep rate decreases. We undertake to relate this to the hardening effect. We interrupt the test, unload the test piece, and apply a twisting moment. If the hardening is isotropic, i.e., determined by a scalar parameter, the hardening due to the tension will also affect the creep rate in torsion. In fact, the creep in torsion is the same as for a specimen that was not first subjected to tension. In fact, hardening anisotropy can be observed only when the type of stress state changes during the test. For constant stresses or stresses that vary with a single parameter the theory of creep with isotropic hardening holds, or more exactly, given sufficiently broad assumptions about the nature of the hardening anisotropy, in this case we get a result coinciding with the predictions of the isotropic theory. If the stress state does not change with time, we apply the term quasi-steady-state creep. p. This case is often realized in structures, exactly or approximately. Only for this case do we already have effective design methods.

The basic experimental fact relating to quasi-steady-state creep is that for different forms of stress state the initial sections of the creep curves are similar. This means that if we construct graphs, plotting along the ordinate axis some component of the creep strain tensor or a homogeneous function of the first power of these components and along the axis of abscissas time, then, by changing the scale of the ordinates, we can make all these curves coincide. Therefore the fundamental law of quasisteady-state creep may be written in the form:

$$\dot{p}_{ij} = v_{ij}(\sigma_{kl}). \tag{5.1}$$

If $\dot{p}_{ij}$ is the derivative of the strain with respect to time, we have steady-state creep; for quasisteady-state creep the role of time is played by the argument $\tau(t)$, which takes into account the shape of the similar creep curves. The possibility of representing the experimental data for constant stress in the form (5.1) must be regarded as the definition of quasisteady-state creep, i.e., the delimitation of the class of materials examined below and their conditions of service. As is known, in the general case, for isotropic material the tensor v_{ij} is expressed in the form of a linear function of three tensors: the unit tensor, the tensor σ_{ij} and the square of the tensor σ_{ij}. The coefficients of this expression, in their turn, are arbitrary functions of the three invariants of the stress tensor. The use of this kind of general relation either for interpreting the experimental data or subsequent applications, holds little promise, and the circle of possible assumptions must be narrowed. A significant role in the development of the theory has been played by the idea of the existence of a creep potential, i.e., a stress function $f(\sigma_{kl})$ such that

$$\dot{p}_{ij} = \frac{\partial f}{\partial \sigma_{ij}}. \tag{5.2}$$

There have been numerous attempts to prove the existence of a creep potential, but these will not be considered here. We shall merely note that the existence of a creep potential does not follow inevitably from the principles of thermodynamics and requires additional hypotheses. One of the decisive circumstances is the fact that it is simpler to develop the formal apparatus of the theory of creep in the presence of a creep potential. It is possible to formulate certain variational principles that facilitate the solution of problems; uniqueness theorems can be proved.

Assuming that creep is not accompanied by volume deformation, we conclude that the creep potential depends on the second and third invariants of the stress tensor. If the potential depends only on the second invariant, we arrive at the following equations of creep:

$$\dot{p}_{ij} = \frac{3}{2}\frac{v(\sigma_0)}{\sigma_0}(\sigma_{ij} - \sigma\delta_{ij}), \qquad 3\sigma = \sigma_{ii} \qquad (\sigma_0^2 = {}^3/_2\,(\sigma_{ij} - \sigma\delta_{ij})(\sigma_{ij} - \sigma\delta_{ij})). \tag{5.3}$$

The quantity σ_0 is called the stress intensity.

Relations of type (5.3) are well known in the theory of plasticity and were transferred to the theory of creep by analogy. They form the basis of most existing theories. By analogy with theories of plasticity of the St. Venant type, given the associated law of creep it is possible to construct creep equations of the form:

$$\dot{p}_1 = -\dot{p}_3 = v(\sigma_1 - \sigma_3), \qquad \dot{p}_2 = 0 \tag{5.4}$$

where $\sigma_1 > \sigma_2 > \sigma_3$ are the principal stresses.

A more general assumption is that the creep potential depends on some equivalent stress $\sigma^* = \sigma_0 g(\theta)$, where θ is the so-called angle of similarity of the deviators, i.e., the angle formed by the vector of the octahedral tangential stress with the projection of one of the principal axes on the octahedral plane. The choice of σ_0 and θ as independent invariants is completely equivalent to the usual choice of a system of invariants of the deviator of the stress tensor [51]. The vector of the octahedral shear rate generally forms a certain angle with the vector of the octahedral tangential stress; we shall denote the tangent of this angle by $\varkappa(\theta)$. We resolve the vector of the octahedral shear rate into two components: one coinciding in direction with the vector of the octahedral tangential stress and the other perpendicular to it; these we denote by v_s and v_t, respectively (correct to a constant multiplier). Then from (5.2) and the above assumption we get:

$$v^* = \frac{v_s}{g} = v(\sigma^*), \qquad \frac{v_t}{v_s} = \varkappa(\theta) = -\frac{g'(\theta)}{g(\theta)}. \tag{5.5}$$

In the special case where the creep law (5.3) holds true, $\varkappa = 0$ and v^* is the intensity of the creep rates. Equations (5.4) may also be derived as a special case. The direct verification of (5.5) is the simplest means of clarifying the role of the third invariant in the law of three-dimensional creep. Almost all the published data on creep in the complex stress state have been analyzed in this way. The deviation of the quantity $\varkappa(\theta)$ from the predictions of the theory is a very sensitive criterion that enables us to estimate its correspondence with reality. In principle, we can use (5.5) to determine the function $g(\theta)$ if $\varkappa(\theta)$ has been found.

Actually, the considerable scatter of the experimental data in every, even the most careful, investigation makes it impossible to do this reliably; however, certain conclusions are possible. The most complete and systematic investigations of creep in the complex stress state are those made by Johnson et al. [52]. Since it is impossible to dwell on the technical details, we shall merely draw attention to their extreme care and accuracy. Complete results have been published for carbon steel at three temperatures, for aluminum alloy at two temperatures, for magnesium alloy at two temperatures, and for heat-resistant nimonic alloy at one temperature. The tests lasted 150 hours; thus, the first period of creep was investigated. In analyzing the results, use was made of the above-mentioned similarity between creep curves. The individual curves were reduced to a certain standard mean curve and a comparison made of the creep rates with respect to a certain function of time.

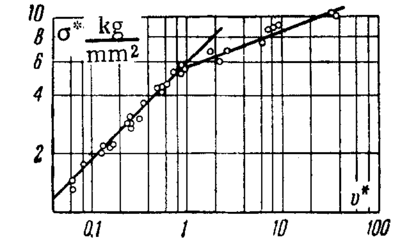

Fig. 11.

We present the data for 0.17C steel at 450°. In Fig. 10 we have plotted the points corresponding to the experimental values of $\varkappa$. If $\varkappa = 0$, then Eqs. (5.3) hold true. Relations (5.4) correspond to the dashed curve on the graph. Clearly, the experimental points are grouped close to the axis of abscissas; most of them lie below the axis, but only just, and it is impossible to detect a definite law. Evidently, it is a question of the random scatter typical of creep tests. However, four points, relating to the highest stresses, constitute an exception. These lie close to the dashed curve derived from (5.4). The satisfactory confirmation of one of the conclusions following from (5.3) obliges us to put $\sigma^* = \sigma_0$; thus, it remains to verify the dependence $v^*(\sigma_0)$.

The quantity v^* is now equal to the intensity of the strain rates. Figure 11 shows this relation in log-log coordinates. It is fairly well represented by two segments of straight lines, i.e., we get a power law with one exponent at low and another at high stresses.

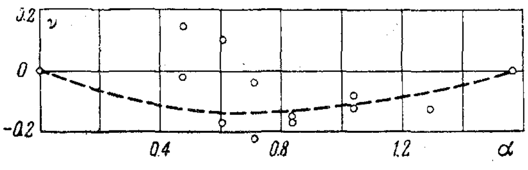

Fig. 12.

Using the exponential law of creep, we can also detect a discontinuity in the graph plotted in semilogarithmic coordinates, but this discontinuity is less abrupt. If so desired, it can be ignored. It should be borne in mind that all four experimental points relating precisely to those tests for which the value of $\varkappa$ is close to the value predicted by formulas (5.4) lie in the region of high stresses. Thus, at low stresses the creep is well described by Eqs. (5.3), while at high stresses the best results are given by Eqs. (5.4). The shortage of experimental points makes it impossible to carry out a reliable verification of relations (5.5) for the region of high stresses, where $g(\theta) \neq 1$. Analogous results are obtained for other materials and temperatures.

As a second example, we present an analysis of the data of experiments on 1Kh18N9T austenitic steel at 600° [53], where a comparison was made between the rates of steady-state creep. These tests lasted considerably longer – up to 2000 hours. Figure 12 shows the dependence of $\nu = g'(\alpha)/g(\alpha)$ on a certain quantity α (the tests were for combined torsion and tension – stresses τ and σ, respectively – $\operatorname{tg}\alpha = \sqrt{3\tau/\sigma}$), which is a function of θ. In principle, this method is no different from that described above. The advantages in any particular case are associated only with the form in which the initial experimental data are presented. According to (5.3), we should have $v = 0$. The dashed curve shows $v(\alpha)$ according to (5.4). The scatter of the experimental points is fairly large. In general, the experimental points lie midway between the values predicted by the two theories, but with a bias toward (5.4). Figure 13 shows v_0 as a function of σ_0, in accordance with (5.3), and Fig. 14 v^* as a function of σ^*, in accordance with (5.4).

In the first case the points are spread over a fairly broad band. The left-hand curve relates to tests in simple tension, the right-hand curve to torsion tests; the intermediate points correspond to combined stress states. In the second case the points corresponding to tension are again isolated; the torsion curve now lies much closer than before. The points corresponding to intermediate stress states are grouped quite close to the torsion curve, without a definite tendency to concentrate on either side. Clearly, for this material Eqs. (5.4) give a better result than Eqs. (5.3). The isolated position of the points corresponding to simple tension is evidently connected with the technical conditions of the experiment.

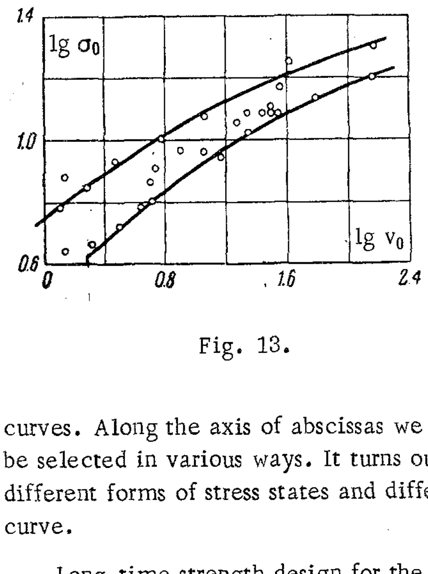

Fig. 13.

The majority of experimental investigations of creep in the complex stress state relate to round specimens in combined tension and torsion, the range of angles θ extending over 30°. If we assume that the material is isotropic and has the same properties in tension and compression, this is sufficient; however, it is not sufficient to permit conclusions about isotropy or anisotropy. The same effects can be attributed both to the third invariant, i.e., the function $g(\theta)$, and to the anisotropy of the material. Therefore a careful, independent check on the isotropy of the material is absolutely essential.

The following problem of the theory of the complex stress state relates to the criteria of failure. The most complete experimental data pertaining to this problem are again those of Johnson [52]. Apparently, for the majority of materials the criterion of long-time rupture is the maximum normal stress. Suppose that we carry out tests to destruction for every possible form of stress state. The results of these tests are presented in the form of ordinary long-time strength curves. Along the axis of abscissas we plot the rupture life and along the ordinate axis some equivalent stress, which may be selected in various ways. It turns out that if we take σ_{max} as the equivalent stress, then the points corresponding to different forms of stress states and different stress levels will lie on a single curve.

Long-time strength design for the complex stress state is now based almost exclusively on the maximum normal stress. Clearly, the creep rate is determined by the stress intensity or the maximum tangential stress. Therefore, depending on the type of stress state, failure may occur at values of the strain varying within very wide limits. Under conditions of stress concentration, when the maximum normal stress is large, we get intense cracking without appreciable creep, and fracture may be brittle in character with relatively short applications of the load [54]. Copper test pieces with a stress concentrator suffer brittle fracture after 1-2 minutes, whereas smooth specimens tested in tension exhibit purely viscous behavior.

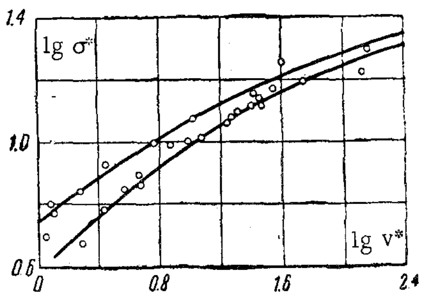

Fig. 14.

In [55] the criterion of maximum normal stress was somewhat refined for alloy 437B and one type of austenitic steel. It was shown that more accurate results are obtained if we take as the equivalent stress

$$\sigma_{eq} = \lambda\sigma_{max} + (1 - \lambda)\sigma_0. \tag{5.6}$$

In subsequent publications Johnson has pointed out that, in addition to the first group of materials, failure of which is governed by the maximum normal stress, there is a second group for which the criterion of long-time strength is the quantity σ_0. The following features of these two groups have been noted.

Group 1. Crack formation begins in the early stages of creep, the cracks being more or less uniformly distributed over the volume of the material. During the creep process the cracks grow in size and their number increases. In the first and second stages the creep rate depends on σ_0, as follows from the theory, but in the third stage it depends on σ_{max}.

Group 2. Crack formation is not observed during creep; cracks appear immediately before rupture and are strictly localized in the area of eventual failure. In all three stages the creep rate depends on σ_0. It might be assumed that the second group suffers viscous fracture, but this is not so. Transition to the third stage and rupture are accompanied by very small strains.

The phenomenological theories of creep, including the description of the process of failure, are still in an early stage of development. Attention is drawn to Kachanov's book [56], in which it is suggested that cracking does not affect creep and the process of failure is described as the progress of a failure front, behind which the material has already lost its carrying capacity, whereas in front of it the stress distribution is determined by the law of creep. This scheme is particularly applicable to materials of the second group. For materials of the first group it is necessary to take into account the effect of cracking on the creep rates and their distribution. An attempt to construct a theoretical framework for this case is described in [57].

In this review it has not been possible to dwell on a number of important details and several relatively uninvestigated areas have been ignored. Recently, strenuous efforts have been made to reconcile the viewpoints of specialists in metal physics, on the one hand, and those concerned with the theory of creep and its engineering applications, on the other. The above presentation of the phenomenological viewpoint was prompted by a desire to draw attention to the factual material already accumulated in the literature and still requiring analysis.

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